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Chapter 1 Introduction

Why This Book ?

My interest in reinforced concrete began during the construction of Winterholm, a solar heated house-greenhouse complex of my own design, built into a steep south-facing hillside near Montreal, Canada. Winterholm is partially earth-sheltered, its different levels following the hillside upward stepwise. Its total footprint is 4900 ft^2. The outer shell of the building and most interior components are constructed of reinforced concrete. Winterholm contains examples of all of the structural members described in this book.

I had originally engaged a structural engineer to "detail" the Winterholm rebar but, after consulting literature on the subject, I realized that for a project of this size the design process was not "rocket science" and took over the task myself, with the help and encouragement of Prof. David Selby, P. Eng. of McGill University. In so doing, I also recognized that someone lacking a technical background and timely expert advice would have found the same task very difficult. I reasoned that I could use this experience for the benefit of other builders by making available the knowledge I have gathered in designing the various structural units of Winterholm. As Winterholm nears completion, Mathieu Roberge, P. Eng., who holds degrees in both mechanical and civil engineering, has been a valuable addition to the project.

As a building material, reinforced concrete offers many advantages but its use in small projects has been restricted by several factors, the high fees of structural engineers being one. The worst case scenario is the one where a concrete house or an internal component such as a concrete beam is built with rebar that has been sized and positioned by intuition. It is the aim of this book to make building with reinforced concrete more accessible to the do-it-yourself and small project builder and reduce or (hopefully) eliminate rebar designs based upon guesswork. In any situation where concrete is not uniformly supported and is subjected to loads, including its own considerable weight, the location and size of the reinforcement are critical. Chapter 3, which deals with the analysis of loads (stresses), provides information leading to the correct placement of steel reinforcement. In Chapter 5 this teaching is combined with the bending formulas developed in Chapter 4 to produce a complete rebar size/placement plan.

With the exception of footings and slabs-on-grade, the time spent on form construction will greatly exceed that required for assembling rebar cages or grids. Accordingly, a large portion of this book is devoted to forming and forming problems. The methods described (the "Winterholm" designs) were specifically developed for reinforced concrete projects with reusable forming materials.

Sources of Design Information

One might reasonably ask from what other sources would information be available for small-scale reinforced concrete construction? Unfortunately current engineering textbooks, including the introductory ones, are intended for students whose educational path will lead toward the design of bridges and skyscrapers – not one or two storey residential houses. Also, a better understanding of the factors affecting the strength and stability of concrete structures has led to an ever-increasing complexity of the design process, usually with the assistance of sophisticated computer programs. To find design literature suitable for small-scale projects we must turn to older publications. In 1943, Prof. Harry Parker published a book entitled "Simplified Design of Reinforced Concrete" (Wiley & Sons). This textbook, specifically intended for builders and architects (i.e. non-engineers), was re-published in several editions and as sections of larger volumes, continuing long after Prof. Parker's passing.

Although the early-edition Parker books provided design information suitable for small reinforced concrete projects, their focus was on structures which were larger and more complex than those considered here. We must also examine the basic objectives of a design engineer, which may not coincide with our own. For the engineer, a reduction in the quantity of rebar or concrete results in financial savings, and the larger the project the greater the savings. Thus it becomes the engineer's objective to minimize the quantity of materials used without rendering the structure unsafe. As material costs escalate, this issue becomes increasingly important. Unfortunately, there is a price to be paid for this fine-tuning, which is increasing complexity of design. Even the Parker books contain sections dealing with the reduction of rebar in certain structural members. Although the design information may be clearly presented, every rebar reduction scheme involves decisions and calculations, increasing the opportunity for error. In this book, simplicity of design and the safety of the structure take precedence over economic concerns. For a non-engineer working with reinforced concrete probably for the first time, a simple conservative design system is optimal. In a small project, a few more dollars spent on steel should not be significant, and extra steel in concrete is never entirely wasted.

Design Systems

Until the early 1960's, reinforced concrete structures were designed according to a system known as *Working Stress Design* (WSD) which, in theory, is based upon the elastic properties of concrete under compression and steel rebar under tension within certain limits. In the latter half of the twentieth century, WSD was superseded by the more sophisticated *Ultimate Strength Design* (USD) system, which can accommodate multiple stresses acting upon a single structural member and loads requiring different safety factors. USD-type design systems are now used by all structural engineers. Unlike USD, WSD does not distinguish between different types of loads, such as the weight of a bridge (static load) and the weight of the traffic it may carry (live load). Using WSD, one must determine the maximum possible combined load and design accordingly. However, for the assumed users of this book, USD offers few advantages and some important disadvantages. If we consider, for example, an earth-sheltered house, the maximum loads from concrete, earth and snow are predictable, and design calculations can easily be made by WSD. Even in their simplest form, USD calculations are more complex than those of WSD, increasing the chances of error. Most USD beams are smaller in cross section than those designed by WSD. Materials (and, of course, money) are saved, but the increased allowable stresses in the concrete may permit *creep*, the slow deformation of concrete by compressive stress, to become a problem. Unlike beams designed using WSD, concrete beams designed by USD may show significant deflection (bending) which, in engineering texts, is discussed under the heading of *serviceability*. The later editions of the Parker textbooks, which teach the use of the USD design system, contain a chapter on serviceability as do modern engineering textbooks. The subject is not mentioned in the early Parker WSD-oriented editions.

All of Winterholm was built using WSD, which is the design system taught in this book. The ceiling above the living and dining rooms, supported by a central concrete beam, weighs about 23 tons. There has been no noticeable deflection or creeping within this or any other structural unit. The project has been visited by several engineers and I have yet to hear a negative comment concerning its structural integrity.

The two storey sections of Winterholm supported by the mat foundations described in Chapter 5 represent the approximate height limit for structures which should be undertaken by the users of this book. Large structures may be subject to stresses other than those produced by the simple loads described in Chapter 3. If there is a question concerning the applicability of the information within this book to any part of a construction project the user should seek the advice of a structural engineer. However, such advice should be confined to design issues and exclude economy of construction.

Forming

The most important reference for concrete structural formwork is the American Concrete Institute publication “Formwork for Concrete” edited by M. K. Hurd. This is the premier reference book of the formwork industry, and one of its many editions should be available in most major libraries. As one might expect, much of this publication is devoted to major projects, but a large quantity of generally useful information is included. Of greatest importance to the small project builder is the book’s excellent data on the strength and properties of the lumber used to contain wet concrete in forms together with very useful form design tables. The problems which can arise from poor form construction or support and improper concrete pouring procedures are discussed and illustrated by example. While the failure of wall forms will usually result in an incredible mess, form failure during the pouring of an elevated slab can result in injury or death. Concrete forming information is also available in some issues of “Fine Homebuilding” magazine, which also contains other concrete-related articles, such as concrete placement, finishing and curing. Back issues are conveniently available from The Taunton Press on DVD-ROM. In designing the Winterholm forming systems I have used information from the above sources and taken particular care to insure that the elevated slab safety issues described by Hurd were properly addressed. Standard wall form plans have been modified to allow both forms and rebar to be assembled in a logical stepwise manner. Winterholm-design form walls have excellent wind stability and overall rigidity.

For small-scale reinforced concrete work, there are few alternatives to site-built forms. Rental basement wall forms, designed for rapid assembly and quick turnover, are totally unsuited for reinforced concrete projects. Insulated concrete (ICF) forms may allow both horizontal and vertical rebar to be added during assembly. However, use of the ICF system is limited to exterior walls.

The Winterholm forming systems described in this book are designed to be used where a reinforced concrete building is to be built in sections (increments), with forming materials recycled between the different stages. One should begin by dividing the entire project into "construction units" – parts that logically should be poured at the same time. Using large construction units will accelerate the job but require more materials. The building layout should suggest obvious sizes for these. In Winterholm, the largest construction unit was the "wing" – two adjacent rooms totalling 15 x ~35′, divided by a concrete wall. An accurate estimate of the largest construction unit allows the required forming materials to be estimated, but a generous excess should be included. The forming requirements for walls and elevated slabs are, of course, quite different.

Hurd estimates the cost of forming to be 35 – 60% of the cost of a concrete structure. Much of this will be for labour and single-use forming materials. After the initial materials have been assembled, I would estimate the cost of non-reusable forming materials under the Winterholm system as less than 2% of the total cost of a completed construction unit. I am not including plywood losses, as plywood which has deteriorated to the point where it is no longer suitable for forming walls or elevated slab bottoms can be recycled as coverings for platforms and walkways, wall insert and slab-edge forms, soffit form supports and, finally, wall-form alignment tabs. Almost no large sheet plywood is discarded.

A list of Winterholm forming materials is presented in Appendix 2. In planning a project, one should foresee what structural materials other than concrete will be required, and consider purchasing these in advance if they could be useful during forming. The Winterholm plans included a garage attic floor and an upper level deck, both built of wood. Early purchase of their 2x8 floor joists allowed these to be used as *walers* (sometimes called *wales*), the horizontal members of wall forms, and as beams supporting elevated slab forms. The tempered glass sheets which make up the Winterholm roof are supported by sections of 3x2″ angle iron, 1/4" thick, which is sold in 40′ lengths. Some of these were cut to serve as one-piece walers or roof beam supports. Others were cut to frame doorway forms. In the overall construction scheme these were zero-cost forming materials.

Labour and Execution

Building reinforced concrete structures is the domain of large contractors. Unless business is “slack” most of these will not be inclined to divert resources from their major projects to a small client and their minimum fee might well exceed one’s total construction budget. People look at Winterholm, assume I had it built, and conclude that I must be a multi-millionaire. Not true. Winterholm was built in increments – one construction unit at a time and by rarely more than two workers. The assembly of plywood wall forms and long walers requires two workers, as does the placement of support beams under elevated slabs and the installation of heavy rebar beam cages. More than four people on one job tend to get in each other’s way and everyone wants to use the same tools. However, a “gofer” is always a useful addition. Novice workers are assigned interesting tasks, such as cleaning plywood panels and coating these with form-release agent. During concrete pours, workers of all abilities are welcomed. In addition to the concrete pump operator a six-person placement crew is often needed for slab pours. The cost of the concrete pump is an item which requires attention during the pour-planning stage. These are rented with a four-hour minimum charge, which costs about the same as 5 m^3 (6.5 yd^3) of concrete. The pump will also “waste” about 1/3 m^3 (~1/2 yd^3) of concrete which cannot be recovered from the pump bucket by pumping. One should always plan to make maximum use of every visit by a concrete pump.

Building incrementally imposes a limit on the rate at which money can be spent, and this can make even a fairly large project affordable. However, it is essential to secure the approval of the appropriate municipal authorities when the project is still in the planning stage. Building permits are normally valid for short periods only, such as six months. Otherwise, special arrangements must be negotiated. Our Winterholm project agreement requires that written progress reports be submitted at regular intervals. If a building exceeds a certain size or value, one may be required to have an architect and/or a structural engineer “sign off” on the plans. This can be a major expense.

This book should still be of value to someone intending to employ a structural engineer, as it can provide an independent overview of the design process. Even with the design plans set, during construction questions can arise concerning the stability of incomplete structures. In Winterholm, my calculations alerted me to the need for extra rebar to stabilize an elevated slab which could only be partially completed. Also, on small projects lacking cost control supervision, engineers have been known to protect their position by over-designing – at the client's expense, naturally. One should also be aware that most engineers lack hands-on building experience which can be reflected in their designs. The original Winterholm structural engineer designed exterior walls which would have been entirely satisfactory if completed but were impossible to pour. Also, concrete was "saved" by designing the outer wall of the indoor swimming pool with vertical ribs – a forming nightmare. Hurd also provides an example of this type of false economy in a large project.

The stability of reinforced concrete structures is dependent upon the accurate placement of steel rebar within the concrete. On major projects, rebar sections are assembled by experienced workers who are trained to carefully follow engineering diagrams. The work of less competent crews must be closely supervised. The knowledge contained within this book is essential in this regard.

Math Tutorial

Rebar "detailing" requires an ability to apply algebraic formulas. However, I cannot agree that knowledge of algebra should be a prerequisite for the use of this book. Not every builder has studied algebra, or familiarity with the subject may have been lost through lack of use. The Appendix 1 Math Tutorial should enable someone with a good understanding of basic arithmetic, and the patience to study the explanations and examples provided, to have full access to the knowledge contained in this book. The objective is to provide the reader with the math tools needed to handle all of the essential design and analytical calculations.

The tutorial is not an algebra course, and it does not prepare the user to fully comprehend the math used in the development of the Working Stress Design theory described in Chapter 4. For this reason, a detailed explanation is given of each step taken in its development. It is far more important to understand the explanatory text than the accompanying math. In order to safely and intelligently apply WSD (or engineering data of any kind), it is absolutely essential that the theory upon which it is based be fully understood. The correct and safe execution of a reinforced concrete project will require the entire combined knowledge contained within this book. Although nothing need be committed to memory, a thorough understanding of the entire text will allow quick access to needed information.

If, after studying the math tutorial, the reader still does not feel entirely comfortable in applying the essential (numbered) algebraic formulas found in Chapter 4 or elsewhere within this book, then he or she would be well advised to proceed no further without seeking assistance. It would also be prudent for anyone who felt the need to use the tutorial to have his or her initial set of calculations examined by someone familiar with the application of algebraic equations. Important calculations should always be repeated before a design is finalized. Numerical errors are best avoided by repeating design calculations in reverse. As an example, I have included a set of reverse design calculations for the Winter Garden slab in Chapter 5. Errors in reinforced concrete construction may be almost impossible to correct and, because of the large weights involved, could be very dangerous.

Chapter 2

Properties of Steel and Concrete

We can begin by considering the types of loads {stresses} to which building materials can be subjected and examine how concrete and steel react to these stresses, which are illustrated in Fig. 2–1. Fig. 2–1(a) shows a standard-size concrete cylinder being compressed by a hydraulic press, which is how the compressive strength of concrete, measured in psi (lb/in^2) or MPa (Mega Pascals), is determined. Tensile stress acts in the opposite direction, with objects being pulled apart or stretched, as is the steel wire in Fig. 2–1(b). In later discussions, it will be shown that shear in concrete is much more complex than the simple scissoring action in Fig. 2–1(c). Table 2–1 lists the abilities of concrete and steel to resist the different stresses. The most important item is the weakness of concrete in tension (under tensile stress). As one might suppose, in reinforced concrete design, steel (rebar) rods are cast into concrete in areas where tensile stresses are predicted to develop. The identification of zones of tensile stress within concrete structures and the sizing of the required steel reinforcement are major parts of the design process.

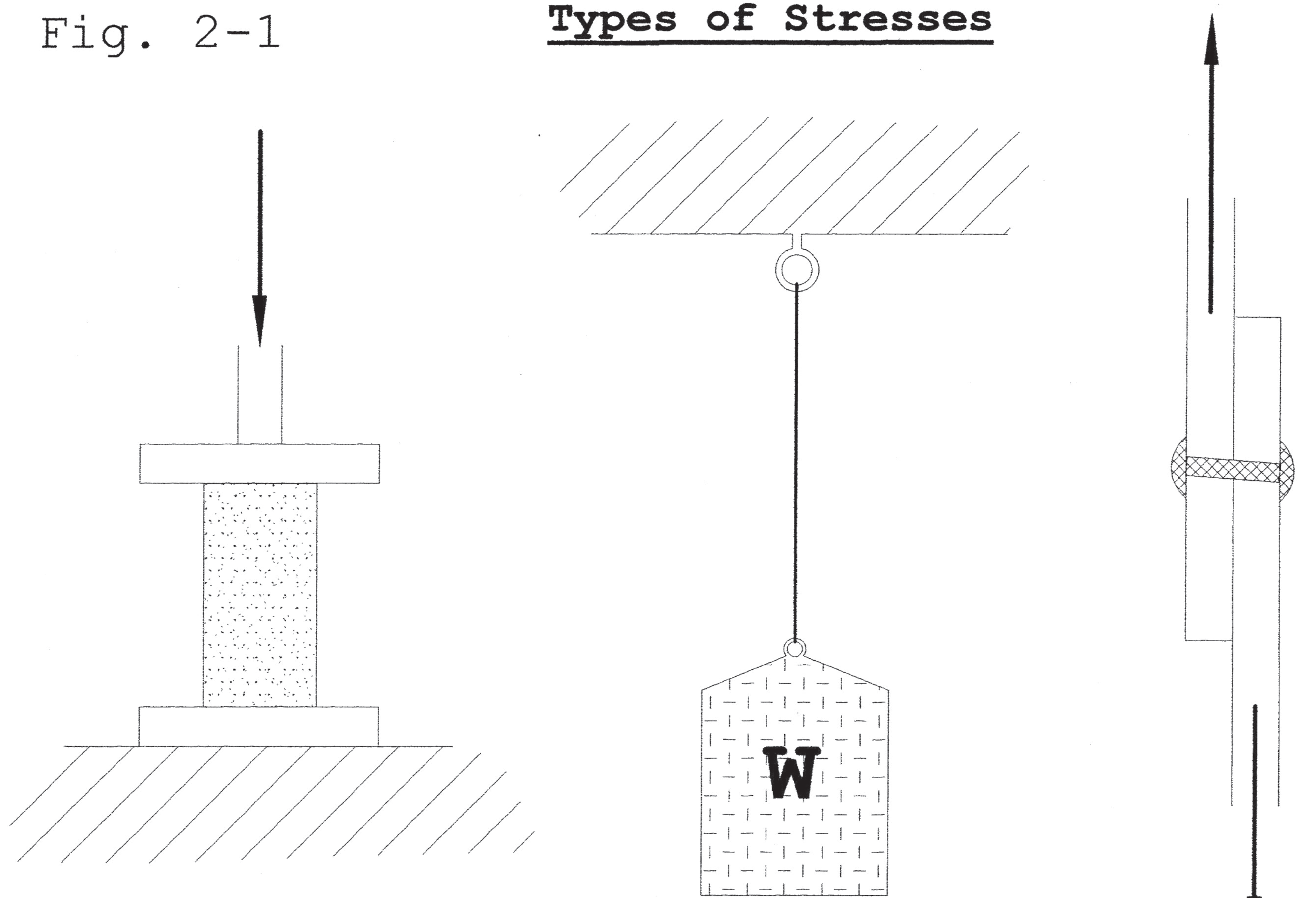
Concrete contains a large number of minute cracks which have little effect upon its ability to resist compressive stresses but greatly decrease its tensile strength. Although in small samples, the tensile strength of concrete is about 10% of its compressive strength, the contribution of concrete to the tensile resistance of larger structural members is small and inconsistent. The tensile strength of wood (lengthwise) is much superior to that of concrete. In this book, the tensile strength of concrete is assumed to be zero, and does not enter into our calculations.

Plain Concrete – Use and Limitations

Unreinforced concrete (*plain concrete* in engineering terms) is the material most frequently used in the construction of residential basements and foundations. In the vast majority of

Fig. 2-1

Types of Stresses



(a) Compression
Of Standard 6" x 12"
Concrete Test Cylinder

(b) Tension
On Wire

(c) Shear
On Rivet

Table 2-1 **Resistance of Concrete and Steel To Different Types of Stresses**

Stress	Compression	Tension	Shear
Concrete	Strong	Weak	Moderately Strong
Steel	Very Strong	Very Strong	Very Strong (N/A)*

*(N/A) - Not Applicable. See Text.

cases the performance of these structures is entirely satisfactory. Plain concrete foundations function well where they carry a fairly evenly distributed load and rest upon a substrate (soil, sand or gravel) which provides uniform support. However, when these conditions are not met, foundation sections can become subject to tensile stresses, resulting in failure by the development of one or more cracks in the foundation walls.

As an example consider, as in Fig. 2-2, a typical residence built on or close to a hillside and on soil containing swelling clay minerals. At the time the house was built the foundation was uniformly supported by the soil. Subsequently, however, a period of drought reduced the soil moisture content under the downhill portion of the foundation, causing the clay soil in that area to shrink and remove support from beneath the foundation footing. This forced the basement wall to act as a cantilever beam (see Chapter 3) attempting to bear the weight of the unsupported concrete section and whatever load it was carrying. As explained in Chapter 3, the upper portion of this type of beam is subject to strong tensile stresses, which caused the basement walls to fail. The unsupported section tilted downward until it was once again supported by the soil. This type of failure will, of course, propagate upward into any rigid overlying material, such as brick. In Montreal, where clay-rich soils are common, severe droughts have damaged basements to the point where some buildings have been condemned.

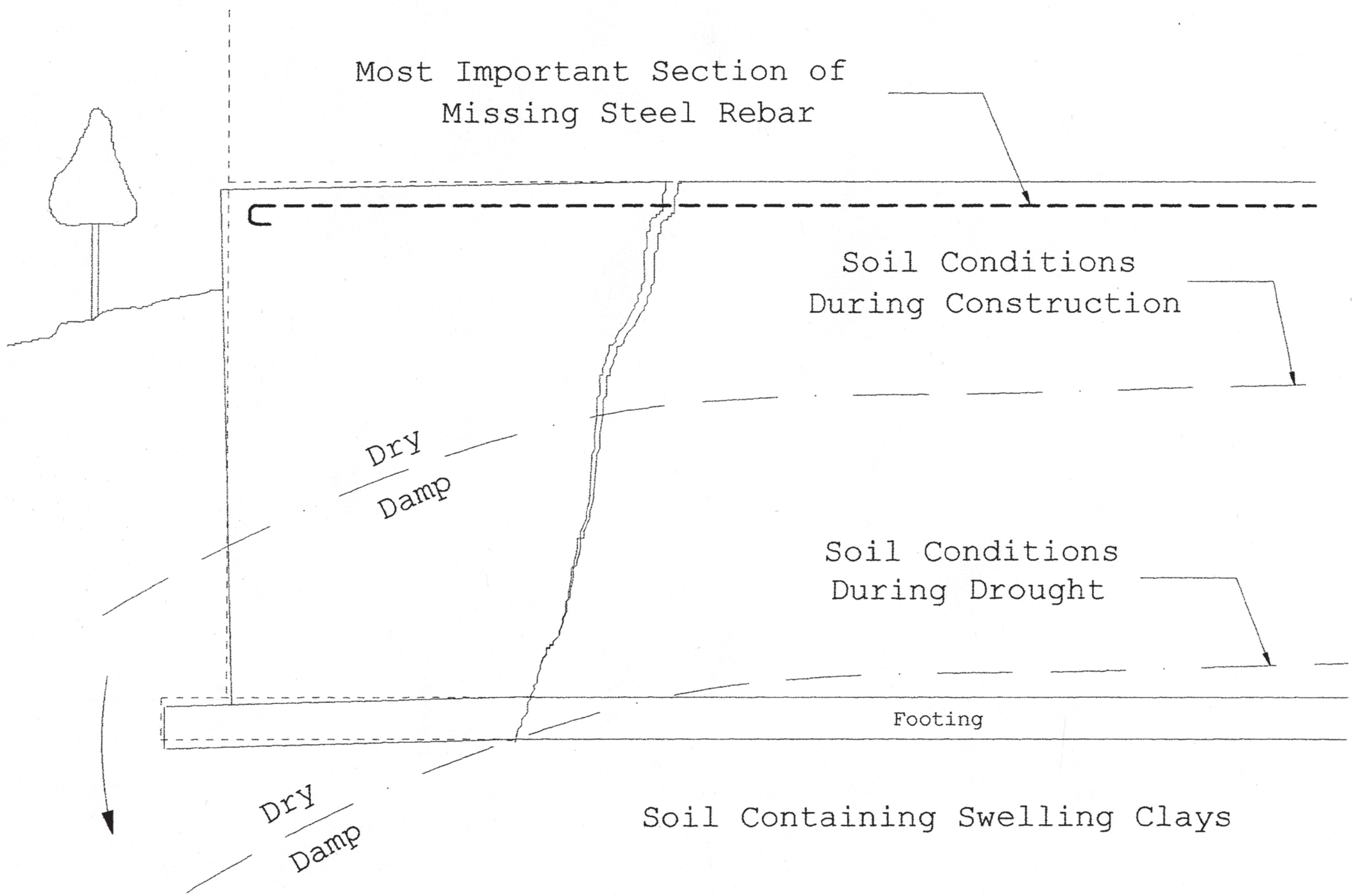
The situation described in Fig. 2–2 is only one of several ways in which plain concrete foundations can be damaged. Deep excavations with drainage, such as sewer installations, frequently result in structural damage to nearby basements. Unequal loading, combined with inadequate footing support, is another frequent cause of foundation damage.

Minimum reinforcement, as described in Chapter 5, would almost certainly have been sufficient to prevent the damage shown in Fig. 2–2. The tensile stresses developed within the concrete wall by the weight of its unsupported section would have been transferred to the steel rebar, and no cracking would have occurred. The uppermost horizontal bars would have been under the greatest stress. Rebar placed only within the footing would not have prevented the wall failure.

The damage seen in Fig. 2-2 (an actual case) was the result of the failure by the builder to correctly evaluate the risks to

Fig. 2-2

Failure of
Plain Concrete Basement Wall



a plain concrete installation posed by an unfavourable combination of soil composition and topography. The cost of installing preventative rebar is much less than that of the engineering required for the repair of basement wall failures. The addition of steel reinforcement to concrete opens the door not only to securing foundations which may not be uniformly supported, but also to the construction of a wide variety of building components, such as beams and elevated slabs, which are impossible to build using plain concrete.

Reinforcing Steel – Descriptive Data

The bars listed in Table 2–2 should meet all of the requirements of a small reinforced concrete building project. 25mm diameter bars were the largest used in constructing Winterholm. Most slabs and walls were reinforced with 10 and/or 15mm bars. Bar selection rules and strategies are discussed in Chapter 5.

The bars listed in Table 2–2 are referred to as being *deformed* because their surfaces are ribbed. This creates a much more secure bond with the enclosing concrete than would be possible with smooth-surfaced bars. Although the ribbing causes the sizes of the bars to be somewhat inexact, in my experience the listings for 10mm and 15mm bars in Table 2-2 accurately represent minimum cross section areas for these bars, so the data in the table can be used with confidence for the important tensile steel (A_s) calculations. The same cannot be said for the larger (20mm and 25mm) bars as measureable bar diameter differences can occur between different rebar consignments. For these, I measure minimum bar diameters in mm using a Metric caliper and then calculate circular cross section bar areas in mm^2. I multiply these results by 0.00155 to find bar areas in square inches. I do not use the approximate dimensions which have been assigned to Metric bars designated by the labels at the bottom of Table 2-2.

Cutting rebar is neither difficult nor time-consuming, so it is economical that it be purchased in the longest available lengths, in order to reduce waste to a minimum. In planning a rebar installation, maximum use should be made of bars which are unit fractions of this length, such as 1/2, 1/3, 1/4, 1/5 etc. of the longest available bars. Where other lengths are required the program “1D Stock Cutter 2.95”, which is available online, can fit up to 100 pieces of varying lengths into the

Table 2-2 **Standard Rebar Sizes**

United States Bars			Canadian Bars*	
Bar No.	And Diam.	Cross Section Area ($in.^2$)	Diam.	Cross Section Area ($in.^2$)
#3	3/8" (0.38")	0.11	10mm. (0.39")	0.12
#4	1/2" (0.50")	0.20	15mm. (0.59")	0.27
#5	5/8" (0.63")	0.31		
#6	3/4" (0.75")	0.44	20mm. (0.79")	0.49
#7	7/8" (0.88")	0.60	25mm. (0.98")	0.76
#8	1"	0.79		
#9	1.128"	1.00	30mm. (1.18")	1.10

*Designated as 10M, 15M, 20M, 25M and 30M by engineers purchased stock with minimum waste (offcut). Including rebar from the next construction phase should further reduce waste. Following this strategy will result in significant financial savings when compared to ordering pre-cut bar lengths from the supplier. In Winterholm construction, only the largest (20 and 25mm) beam bars requiring hooked ends were custom-ordered. If sheltered space is available, short surplus bars should be stacked vertically, allowing selections to be easily made from the surplus inventory.

In ordering long rebar, it is important to specify that, apart from the standard restraints, all bar bundles be strapped or wired about six inches from both ends. This will prevent or greatly reduce bending damage during the loading of the bundles, to which smaller diameter bars are particularly susceptible. Upon receiving an order, all bars should be laid side-by-side on a flat surface, ready for cutting. Bars which are not perfectly straight should be grouped separately, and cut into whatever short segments are required. Bends are easily corrected in short sections, but this is not true of longer pieces.

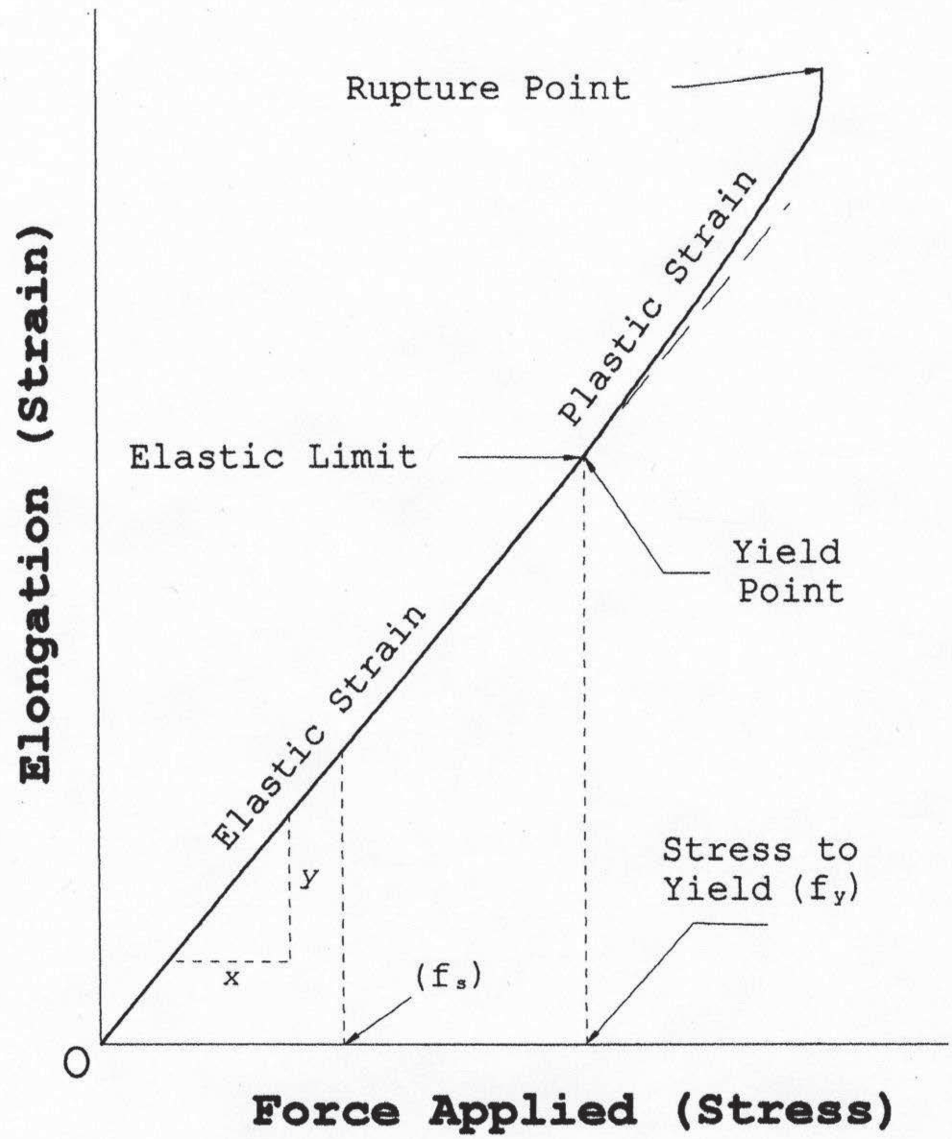
Steel rebar is classified according to the location of its yield point, which is where the straight-line relationship between stress and strain ends on Fig. 2-3. This diagram will be discussed in detail in the following section.

The larger the grade number of a bar, the higher its yield point. A bar of Grade 40 steel, one square inch in cross-section (e.g. U.S. Bar #9), will yield if is subject to a tensile force exceeding 40,000 lb (20 tons), whereas a bar of identical size stamped "60" will withstand 60,000 lb before yielding. According to the design equations developed in Chapter 4, substituting Grade 60 for Grade 40 rebar will result in a 1/3 reduction in the required quantity of tensile steel. Using Grade 60 steel is obviously advantageous where high tensile stresses are anticipated, such as in beams and elevated slabs. However, many components are built to *minimum reinforcement* standards where bar strength is not specified. For these, Grade 40 bars are entirely adequate. In Winterholm construction, only Grade 40 bars were available from the small local supplier.

Another type of reinforcement which deserves mention is welded wire fabric (WWF). This consists of smooth stiff wires of high-tensile steel which have been welded into a rectangular grid

Fig. 2-3

Behavior of Steel Under Tensile Stress



Below the Elastic Limit x/y is a Constant Value
Otherwise Stated:
"Stress is Proportional to Strain"
(Hooke's Law)

and subsequently cut into 4′ x 8′ sections. The most commonly available configuration is 3/16″ diameter wires welded into 6″ x 6″ squares. In Winterholm construction, WWF sheets were added to the swimming pool wall inner rebar layer for maximum crack control.

Reinforcing Steel – Properties

We must begin with a discussion of elasticity, since the elastic properties of steel under tension are a basic component of the Working Stress Design (WSD) system. The most familiar elastic object is the rubber band, which can regain its original length after being stretched. A more useful example is the fisherman's hand-held scale. In this simple device, the weight of the fish either stretches or compresses a steel coil spring, causing an indicator to move down a scale calibrated in pounds. It is important to note that the scale graduations are evenly spaced, with each added pound of fish moving the indicator the same distance. This means that within the limited range of the scale the spring has a linear response to load changes – one identical unit on the scale for every pound of fish added. In nature, many materials behave in this manner under stress, with soft steel being an excellent example.

Fig. 2-3 shows the behaviour of steel (in this case a length of steel wire) under increasing tensile stress e.g., loading as in Fig. 2–1(b). The line shows the stretching or elongation (amount of strain) of the wire under a steadily increasing load (stress). Since there can be no strain without stress, the line must pass through the zero stress – zero strain point on the graph which is the *origin*, shown as "O" in Fig. 2–3. Beyond the origin, each unit of stress, such as the amount "*x*", produces a fixed corresponding increase in strain "*y*". This is similar to each additional pound of fish moving the fisherman's scale indicator the same distance. This fixed relationship between stress and strain is known as **Hooke's Law** which states that the deformation of a body is proportional to the forces causing the deformation or, more simply:

"Stress is proportional to strain"

In Fig. 2–3, the stress vs. strain relationship initially follows the straight line defined by Hooke's Law. However, beyond a certain point known as the *elastic limit*, the data deviate from this line, with increments of stress producing gradually increasing increments of strain until the steel abruptly loses strength prior to rupturing. If the stress is removed before the steel reaches its elastic limit, it will return to its original length, just as the scale returns to zero following removal of the fish. However, beyond the elastic limit the strain is *plastic*, and recovery to the unstressed length is no longer possible.

In Fig. 2-3, f_y is the stress at which the wire or other stretched steel member will yield. f_s is the maximum unit stress in lb/in^2, relative to f_y, which is permitted in rebar by Working Stress Design.

Concrete – Composition

The constituents of "normal" concrete mixes for general use are as follows:

Coarse aggregate
Fine aggregate
Portland cement (type GU in Canada)
Water
Air

Aggregates:
About 60 – 75% of a concrete mix consists of aggregates – clean sand as the fine aggregate and crushed stone as the more abundant coarse aggregate. Not all rock types make suitable aggregates but if one is dealing with a reputable supplier this should not be a matter for concern. Assuming that most reinforced concrete wall pours will be made using a concrete pump, to enter the forms the mix must pass easily through a 4" diameter pipe and flow behind the rebar grids. I use a 14mm (0.55") coarse aggregate mix for wall pours, and may use 20mm (0.79") coarse aggregate for slabs and footings.

Portland cement and water:
Portland cement is produced by heating a finely-ground mixture of limestone and mineral matter containing alumina, silica and iron to near-fusion temperatures and subsequently grinding the resultant clinker to a fine powder. Upon the addition of water to the aggregate-cement mixture, the cement and water form a paste which binds the aggregate mixture into a solid mass as the paste hardens through a chemical reaction between the cement and the water.

Air:
All concretes contain some entrapped air voids. In air-entrained concretes, a huge number of very small bubbles are evenly dispersed throughout the cement-water paste, resulting in an air content within the concrete of about 5% by volume. The advantages of air-entrained concrete, which is recommended for almost all applications, are described in the following sections.

Concrete – Performance

The performance of concrete – ease of placement, durability, permeability to water, and ultimate compressive strength – depend on many factors or, as is often the case, combinations of factors. In some instances the enhancement of one property can be detrimental to another, in which case an intelligent compromise must be made. A prime example is the proportion of water in the concrete mix. The wetter the mix the more easily it can be spread or placed, but adding water will reduce the ultimate strength of the concrete and adversely affect its other desirable properties. Although it is important to understand the factors affecting concrete quality, the major decisions regarding composition of the mix, such as the fine/coarse aggregate ratio, will be made by the supplier. When ordering concrete I need describe it only as: "Air-entrained 30 MPa (4350 psi) pump mix with 14mm coarse aggregate and 100mm (4") slump". The term *slump* is explained in the following sections. In preparing the order, the supplier will proportion the various constituents so that the resultant mix, if correctly placed and cured, will eventually reach a compressive strength of 30 MPa. The performance of concrete can be discussed under the following headings:

Workability:
Workability is the ease or difficulty of placing and consolidating concrete. In wall pours, it is important that the concrete be sufficiently fluid to flow evenly and fill spaces under form inserts such as windows. If correctly placed and vibrated, air-entrained pump mix concrete with 100mm slump is a satisfactory choice for walls.

During Winterholm construction, concrete placement difficulties were encountered when an order was taken directly from a large delivery truck and poured into column forms in extremely hot weather. Heat greatly accelerates the speed at which concrete sets. In retrospect, it would have been advantageous to order that a retardant be added to the mix, causing it to set more slowly. As an alternative to adding water, which weakens the concrete, the fluidity of a mix can be increased by the addition of liquid superplasticizer, a supply of which is commonly carried in the delivery truck.

I now prefer to use site-mixed concrete for small pours. The concrete is placed immediately after it is mixed so it has less opportunity to lose fluidity in hot weather. I control the rate at which it is produced and there is never the problem of having too much or not enough concrete. I pay only for what I need to the nearest 0.1 m^3, unlike the large supply trucks which deliver concrete in 0.5 m^3 (0.65 yd^3) increments.

The best measure of the state of fluidity of a concrete mix, which is referred to as its *consistency*, is its slump. Slump is determined by the *slump test*. A truncated metal cone 12″ high with 4″ and 8″ diameter open ends is filled with fresh concrete and stirred according to a strict regimen. When the cone is lifted the concrete flows laterally and its upper level sinks as the mix “slumps”. The slump measurement is the difference in elevation between the top of the slumped mix and its original height within the cone (12″). The greater the slump the more workable the concrete mix. I have found a slump of 100mm (4″) to be satisfactory for pump mixes. For non-pump mixes for slabs, slumps should be kept to a minimum consistent with workability requirements, 50 – 80mm (2 – 3″) being typical values. Lowering the water/cement ratio in the mix enhances all of the desirable properties of concrete other than workability. Although slump tests are routinely performed on-site at large projects, specifying the desired slump upon delivery should meet the needs of small project builders.

Durability:
Concrete durability refers to its resistance to weathering on exposed surfaces, and in particular to the effects of cycles of freezing and thawing in the presence of moisture. Concretes from mixes with low water/cement ratios show above average durability. However, air-entrainment is by far the most important factor in allowing concrete to resist freeze-thaw damage. The tiny bubbles provide room for expansion during freezing, relieving pressure within the concrete.

Permeability:
To achieve low permeability (water tightness) the concrete paste must itself be watertight. This property is enhanced by a low water/cement ratio in the mix and by proper post-emplacement curing of the concrete. Air entrainment also promotes reduced permeability by allowing for a reduction in the water/cement ratio. Stronger concretes contain more paste, and are therefore less permeable than weaker concretes. The water tightness of concrete can be enhanced by the addition of a sealant to the finished surface. This can be either a liquid or a powder which, when mixed with water, enters and blocks pores on the concrete surface.

Low permeability to water is important for reinforced concrete, especially if ice-melting salts are present. If the concrete cover cannot prevent the rebar from rusting, corrosion products will form expanding sleeves around the rebar. The tensile stresses developed by this process can cause fracturing within the adjacent concrete, and in severe cases, extensive spalling adjacent to the affected steel.

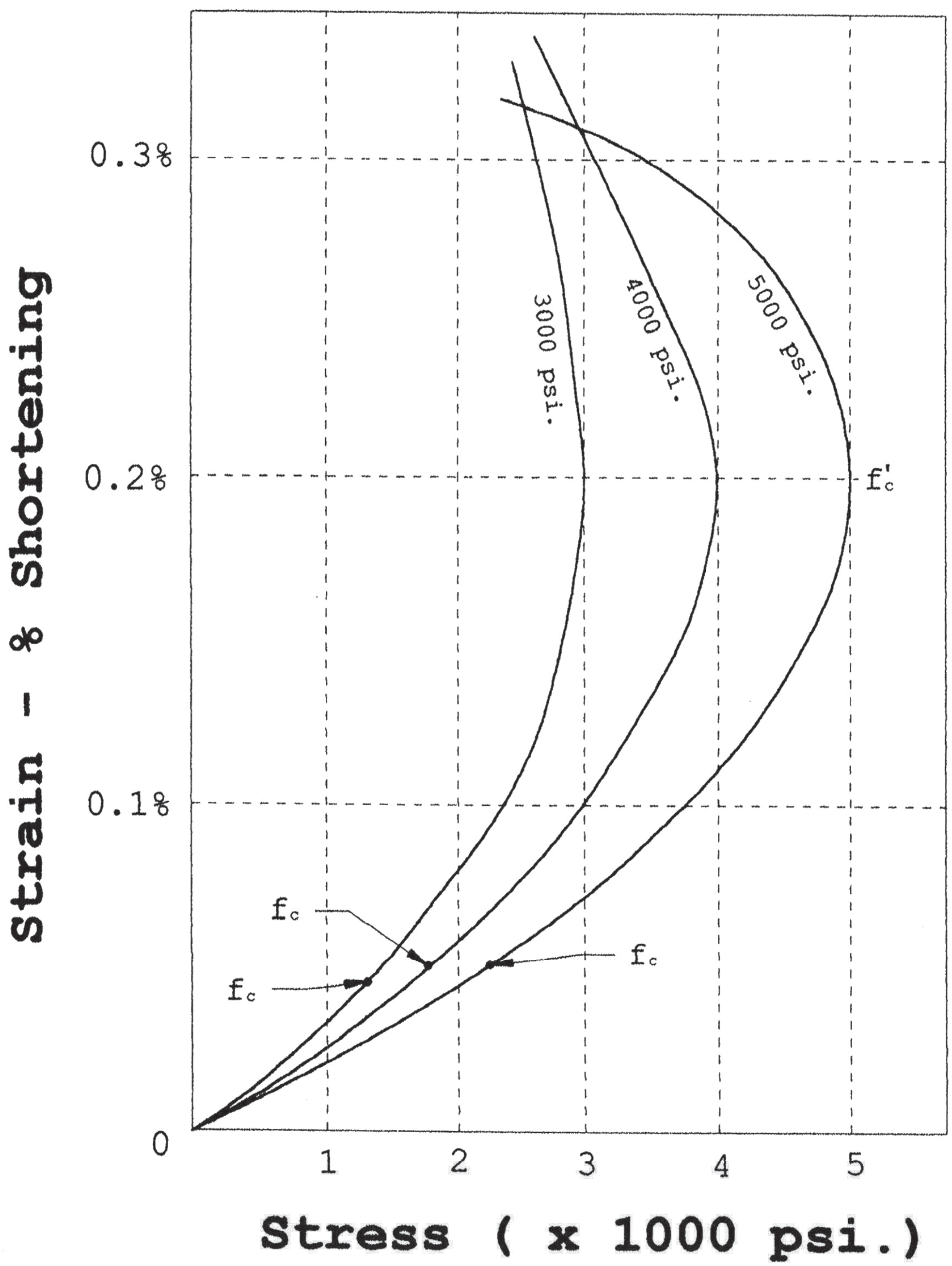
Strength:
Fig. 2–4 shows the behaviour of different strengths of concrete under short-term loading. Concrete is a conglomerate of different materials so its reaction to stress is more complex than that of steel. A near-linear stress vs. strain relationship, approximating Hooke's Law, is evident only in the low-stress portions of the curves. The points labelled f_c in Fig. 2–4 are the maximum unit (lb/in^2 or psi) stresses allowed in concrete under the Working Stress Design (WSD) system. According to the design guidelines developed in Chapter 4, the unit stresses in concretes are never allowed to reach f_c, which improves compliance with Hooke's Law.

Under increasing stress, the curves become progressively non-linear, with all concretes reaching their maximum unit compressive strength f'_c when the uncompressed sample has been shortened by 0.2%. The value of f'_c, which is the compressive strength of concrete specified by the supplier, is determined by crushing a standard test cylinder as in Fig. 2–1(a). The Fig. 2–4 curves show that concrete strained beyond 0.2% grows progressively weaker. Unlike steel, concrete does not have a definite yield strength. In order to obtain meaningful results, the concrete test cylinders used to establish the value of f'_c must be subject to identical conditions prior to crushing. It is specified that they be kept at constant temperature,

Fig. 2-4

Behavior of Commonly Used Concretes

Under Compressive Stress



continuously wet, and that the test be conducted 28 days after they were formed. This is referred to as the standard 28-day strength of concrete and is the value of f'_c listed by concrete suppliers and that shown in Fig. 2-4. In Canada, the compressive strength of concrete is listed by suppliers in terms of Mega Pascals (MPa). This can be converted to the U.S. customary (Imperial) units used in this book as follows:

1 MPa = 145 psi

So for the 30 MPa concrete used in Winterholm:

$$f'_c = 30 \times 145 = 4350 \text{ psi}$$

Canadian concrete is sold by the cubic metre (m^3) so we have:

$$1 \text{ m}^3 = 1.308 \text{ yd}^3 = 35.3 \text{ ft}^3$$

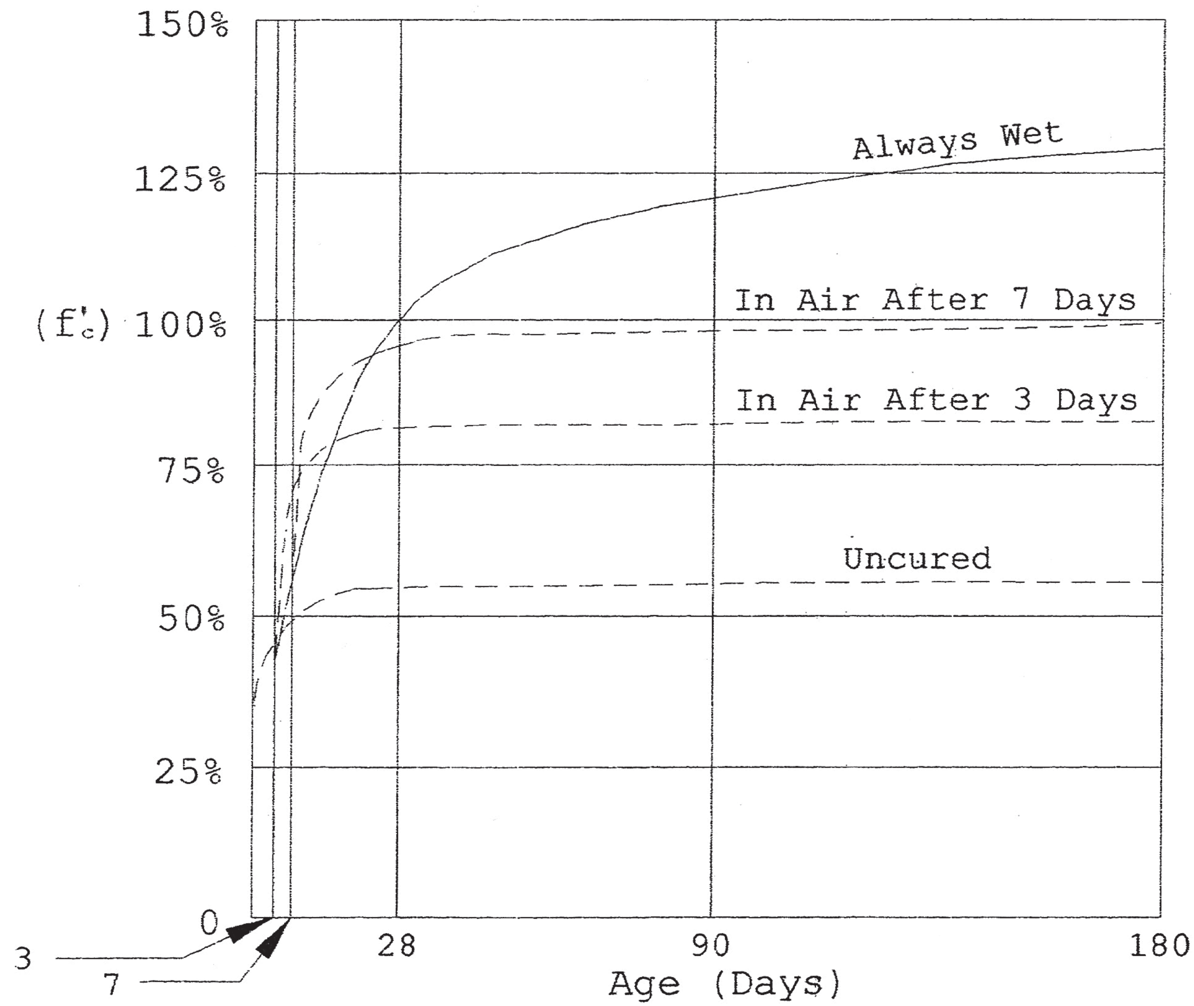
Correct procedures for the placement and vibration of concrete are described in Chapter 6. Although these may have some bearing upon the ultimate strength of concrete, the most important factor by far is the degree to which water is made available to the hardening concrete, which is the process known as *curing*.

The reaction between Portland cement and water, which leads to the hardening of the concrete, proceeds rapidly at first and then progressively more slowly for an indefinite period. As long as water is available the cement paste will continue to harden and all of the desirable properties of the concrete, such as compressive strength, water tightness, and freeze-thaw damage resistance, will continue to improve.

In Fig. 2-5, the 100% line defines the strength of 28 day old "always wet" concrete. The two middle curves show the strengths of concretes which were cured (kept moist) for 3 or 7 days prior to being exposed to drying conditions. The lowermost curve shows the effect of freshly poured concrete being immediately and continuously exposed to drying conditions. It is obvious from Fig. 2-5 that the degree to which concrete has been cured has an enormous effect upon its final strength. Most references suggest a 7-day curing period, after which the concrete should eventually reach its prescribed strength. Being conservative, I usually cure concrete for a minimum of 10 days,

Fig. 2-5

Compressive Strength - Per Cent of 28 Day Moist-Cured Concrete



By Permission - Portland Cement Association and longer if it does not interfere with work in progress. In judging curing time one should take into account the future stresses to which the building component will be subjected. For example, the concrete in beams and elevated slabs should be of the best quality, as it must resist significant compressive stresses. Concrete quality will be much less an issue for interior walls. Fig. 2–5 shows that the standard practice of stripping rental forms from basement walls on the day following the pour produces concrete which is much weaker than the f'_c value purchased, but this loss of strength is concealed by the absence of significant stresses in the concrete.

An impermeable cover should be placed over a freshly-poured concrete slab as soon as its surface becomes accessible. I have found that the thinnest available plastic sheeting gives the best results. It is inexpensive, easily cut, tends to adhere to concrete surfaces, and is often re-usable. Although freshly-poured concrete may look dry, moisture will quickly appear on its surface once the plastic cover is in place. On the morning following the pour additional water is supplied by a garden soaker hose, which is turned on periodically throughout the curing period.

It is usually impractical to cover concrete wall tops as these generally contain large numbers of protruding vertical rebar dowels. On the morning following a wall pour, a hose is used to cover the top of the concrete with water, and this procedure may either be repeated at intervals or replaced by the soaker hose, depending upon how well the form top is retaining the water. The added water normally penetrates down the insides of the forms and their eventual removal reveals a dark, moist wall surface, indicating a curing job well done.

Another factor which will influence the ultimate strength of concrete is the temperature during the pour. As previously noted, the cement-water reaction which causes the concrete to harden is accelerated by heat. Pouring in very hot weather will result in a significant loss of strength by the 28-day cured concrete. On very hot days, avoid pouring beams and elevated slabs, which require the best quality concrete. In cold conditions the opposite is true and strength develops very slowly within the concrete. Concrete can be damaged if it is subjected to freezing conditions within 72 hours of being poured.

Concrete – Laitance

As previously noted, the presence of excess water within a liquid concrete mix will reduce its ultimate compressive strength and adversely affect its other desirable properties. In the absence of a slump test, the telltale sign that a slab mix is “too wet” will be excessive workability (ease of spreading). For wall pours the problem is more difficult to detect. The presence of excessive water in a mix will result in a watery layer appearing on the upper surface of the freshly-poured concrete. Curing reveals an uppermost layer with a whitish milky colour (*laitance*) which consists of a mixture of cement and fine aggregate. The concrete within this whitish layer is extremely weak. Beneath the laitance, concrete quality increases gradually downward but, in thin members, none of the pour is likely to reach the quality specified in the purchase order.

In the case of beams or elevated slabs, where structural integrity depends upon the capacity of the concrete to resist compressive stress and bind firmly to rebar, there may be no alternative other than the removal of the defective concrete by means of a jackhammer. Core samples taken with a diamond drill and analysed by a testing laboratory give the best indication of concrete quality. Alternatively, the *Schmidt Rebound Hammer* offers a less definitive non-destructive (and less expensive) means of assessing concrete quality. This device consists of a small hand-held box containing a spring-loaded steel piston with a rounded tip. The end of the piston is pressed firmly against a smooth area on the concrete surface until the spring is released. The rebound reading given by the instrument is a measure of surface hardness, which varies according to the compressive strength of the concrete. Although this device can only provide an approximate value for compressive strength, it is very useful in determining concrete uniformity and for comparing different concretes. A Schmidt hammer can usually be rented from companies dealing with concrete quality evaluation.

Chapter 3

Mechanics

In engineering, the study of the action of forces upon structures and their components comes under the heading of *mechanics*. As was shown in Chapter 2, external forces (stresses) applied to a body produce changes in its shape (strains). In the reinforced concrete design process, our objective will be to reduce deformations of structural units to the point where the strains are so small that they can be ignored. A conservative WSD system allows us to do this. By contrast, members designed by USD-type systems have planned deformations, such as beam deflections, which require extra *serviceability* calculations. In this book, deflection calculations are confined to steel structural members and to wood formwork in Chapter 6.

The objective of this chapter is to estimate the size and distribution of the internal stresses within structural members, with rectangular beams being the logical subject for investigation. Placing a load on a beam results in the development of internal stresses – compressive, tensile and shear – within different parts of the beam. The resistance of a beam to bending or shear is determined by the ability of its entire body to collectively resist these stresses. In Chapter 4, our knowledge of internal stresses will be used to design structural members which will meet the previously stated "minimum deformation" objective.

The subject matter of this chapter is subdivided as follows:

Reactions: How loads on beams are transmitted to their supports.
Bending Moments: The tendency of loaded beams to bend.
Shear: The tendency towards internal slippage within beams.
Beam Mechanics: The internal stresses within beams.
Restraints: Beam end conditions which modify bending moment calculations.
Cantilever Beams: Beams supported at one end only.
Steel Members: Commonly used items.

Reactions

A loaded beam presses downward upon its supports and the supports must press upward with identical force against the underside of the beam. These upward forces are designated as *reactions* and in Fig. 3-1 are labelled R_1 and R_2. Obviously, the sum of the reactions $R_1 + R_2$ must be equal to the combined weight of the beam and its load. If the load is spread uniformly along the beam, concentrated at its midpoint, or distributed symmetrically relative to the beam midpoint, the reactions R_1 and R_2 will be equal to each other and each will be half the total load. In many instances this will either be the case or be close enough to allow equal weight distribution to be assumed. We must, however, be prepared to deal with situations where the load or loads are unequally partitioned between the reactions.

Let us assume that, as in Fig. 3–1, the supports are 180" apart and a 5000 lb load is placed upon the beam 60" from the left side support. The beam is now *eccentrically loaded* and the reactions are no longer equal. Since the load is closest to the left side support we would expect the reaction R_1 to be greater than R_2 and this is indeed the case. The partitioning of the load between the two supports is determined by the following rule:

"The reaction at a support is equal to the load multiplied by its distance from the <u>opposite</u> support, divided by the length of the entire span."

In Fig. 3–1, the 5000 lb load is partitioned as follows:

$$\text{Reaction } R_1 = \frac{5000 \text{ lb} \times 120"}{180"} = 3333 \text{ lb}$$

Reaction R_2 will, of course, be 5000 – 3333 = 1667 lb
It can also be directly calculated as:

$$\text{Reaction } R_2 = \frac{5000 \text{ lb} \times 60"}{180"} = 1667 \text{ lb}$$

If a beam carries several loads, the above rule is applied to each individual load, and the resultant weight distributions are summed to give the total reactions at the two supports. In Fig. 3-2, one of the loads is a uniform 8000 lb block which extends over 80" of the beam. For this

Fig. 3-1

Reactions: Single Concentrated Load

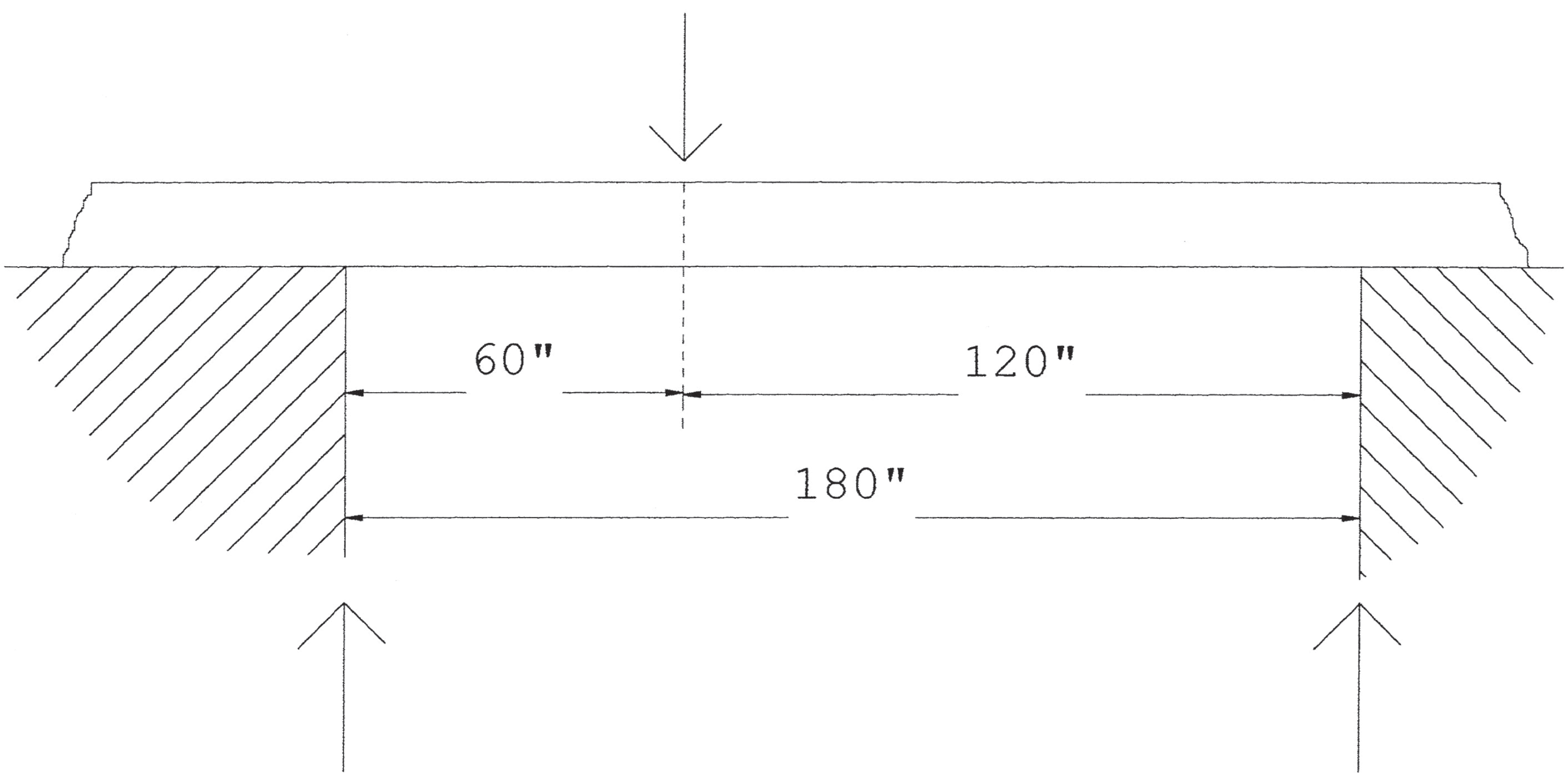
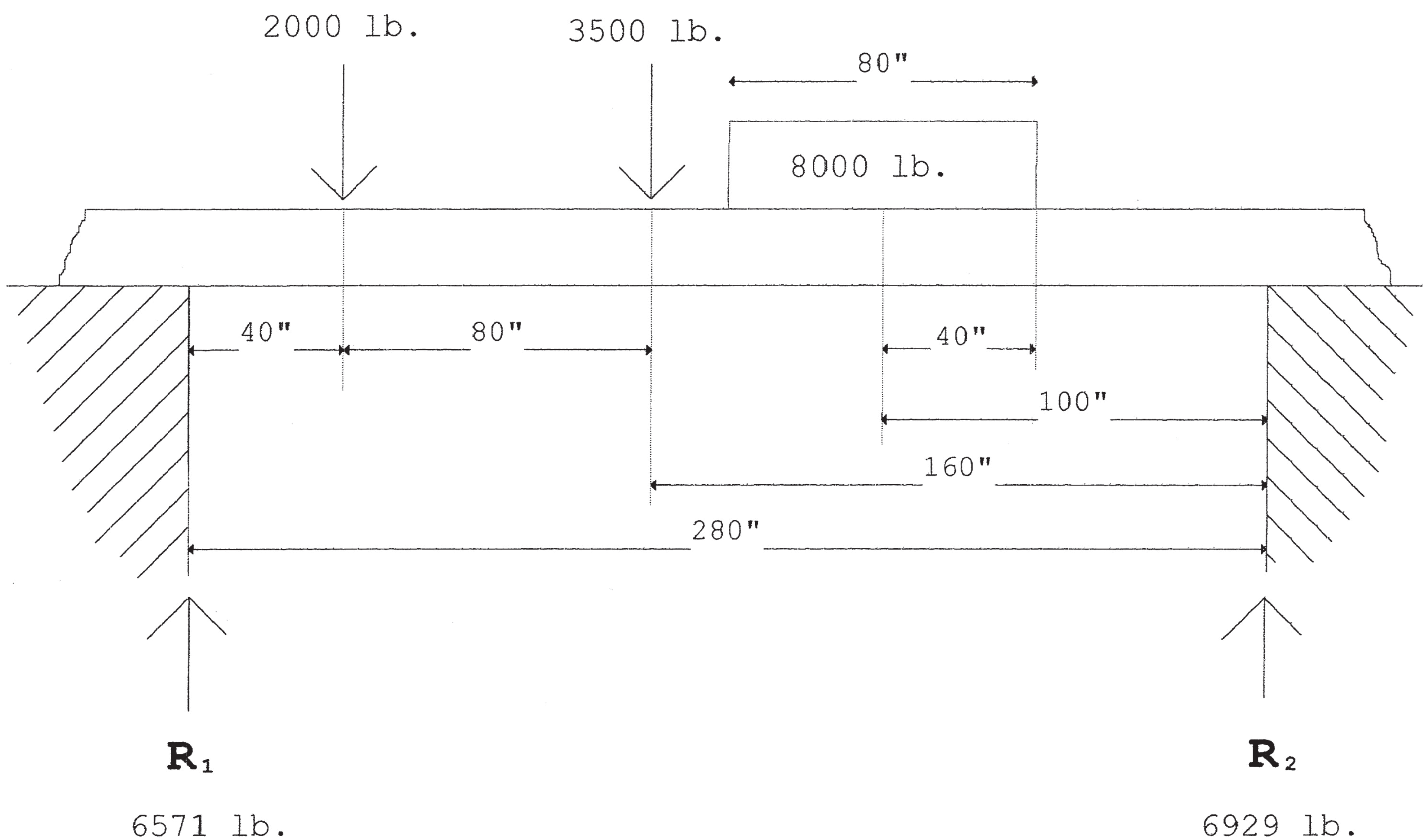


Fig. 3-2

Reactions: Multiple Loads



calculation, the weight of the block is assumed to be concentrated at a single point – its centre of gravity 40" from either end.

$$R_1 = \frac{2000\text{ lb} \times 240" + 3500\text{ lb} \times 160" + 8000\text{ lb} \times 100"}{280"}$$

$$= \frac{480{,}000\text{ lb} + 560{,}000\text{ lb} + 800{,}000\text{ lb}}{280"} = 6571\text{ lb}$$

$$R_2 = \frac{8000\text{ lb} \times 180" + 3500\text{ lb} \times 120" + 2000\text{ lb} \times 40"}{280"}$$

$$= \frac{1{,}440{,}000\text{ lb} + 420{,}000\text{ lb} + 80{,}000\text{ lb}}{280"} = 6929\text{ lb}$$

Check: $R_1 + R_2$ = 6571 + 6929 = 13,500 lb
Total load = 2000 + 3500 + 8000 = 13,500 lb

These calculations will be needed to determine bending moments and maximum vertical shear in eccentrically loaded beams and elevated slabs, and at column tops. They are also generally applicable to all construction where elevated loads are supported.

Bending Moment

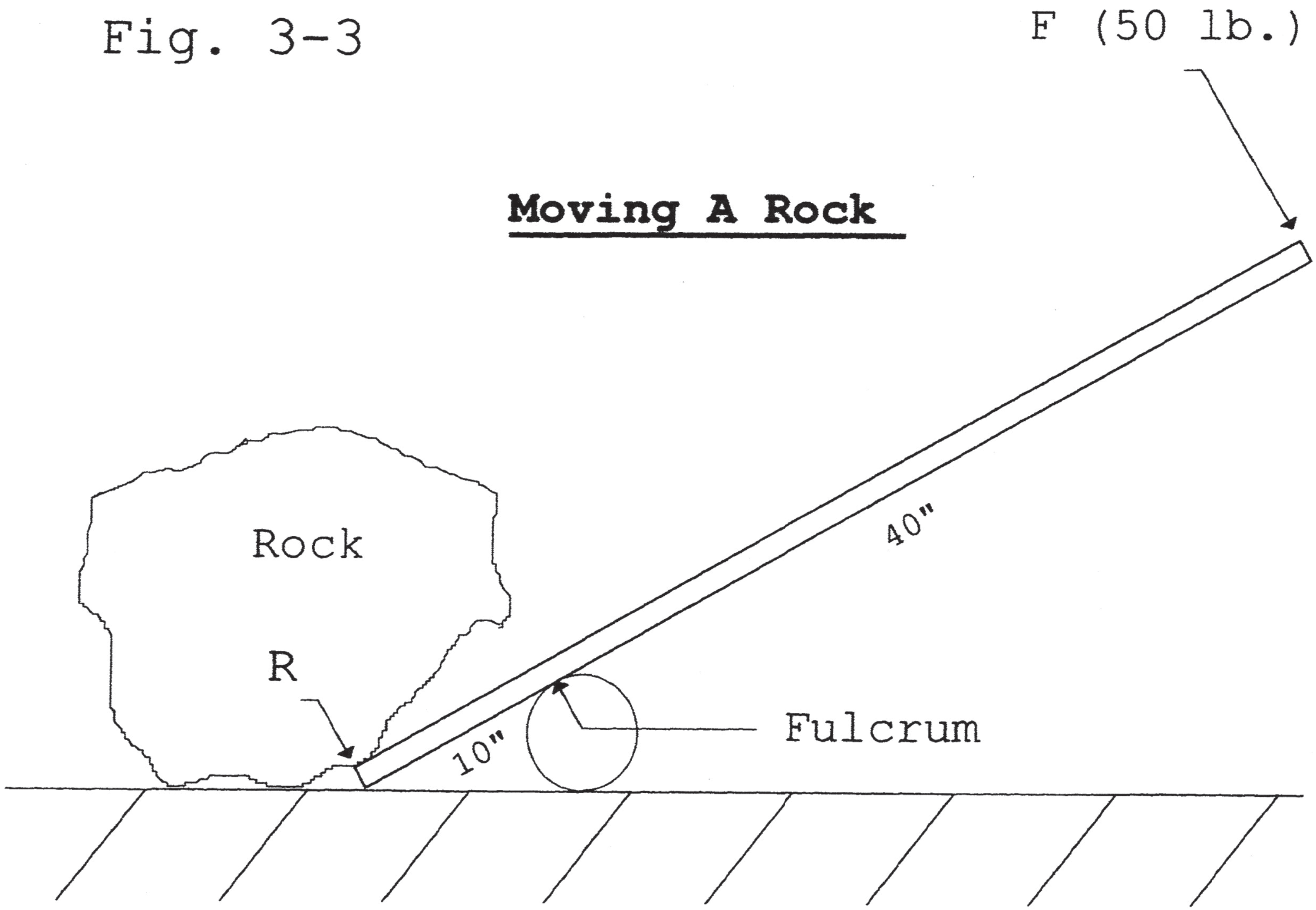
Let us begin by considering what is meant by the term *moment*. Suppose, as in Fig. 3–3, we wish to move a large rock using a 50" steel bar and a round log as a prop (called the *fulcrum*). We know the upward lift on the rock will be increased if:

(1) The fulcrum is placed as close to the rock as possible.
(2) We press downward as far from the fulcrum as possible, i.e. at the far end of the bar.
(3) We press downward with maximum force.

Obviously, the levering operation involves two separate quantities, weights (or forces) and lengths. The variables are the weight of the rock, the force applied to the end of the bar, the length of the bar, and the location of the fulcrum. A moment is defined as a force or weight acting

Fig. 3-3

Moving A Rock



through a given length, which is called its *lever arm*. The levering moment to the right of the fulcrum is the downward force (50 lb), multiplied by its lever arm, which is its distance from the fulcrum (40 in). If we designate the upward levering moment as M_L we have:

M_L = 50 lb x 40 in = 2000 in-lb

Note how the units combine, in-lb meaning a quantity measured in inches multiplied by a quantity measured in pounds i.e., in-lb means inches multiplied by pounds. To the left of the fulcrum the rock presses downward on the bar with a force of R lb which acts through a lever arm of 10 in. The rock's resisting moment, designated as M_R, will be 10R in-lb. If the rock has not moved, the levering system is in equilibrium and the two moments, M_L and M_R, must be equal to each other so:

$$10R \text{ in-lb} = 2000 \text{ in-lb}$$

$$\text{And } R = \frac{2000 \text{ in-lb}}{10 \text{ in}} = 200 \text{ lb}$$

This calculation tells us that the rock was able to resist a lifting force of 200 lb, applied at the bar contact point.

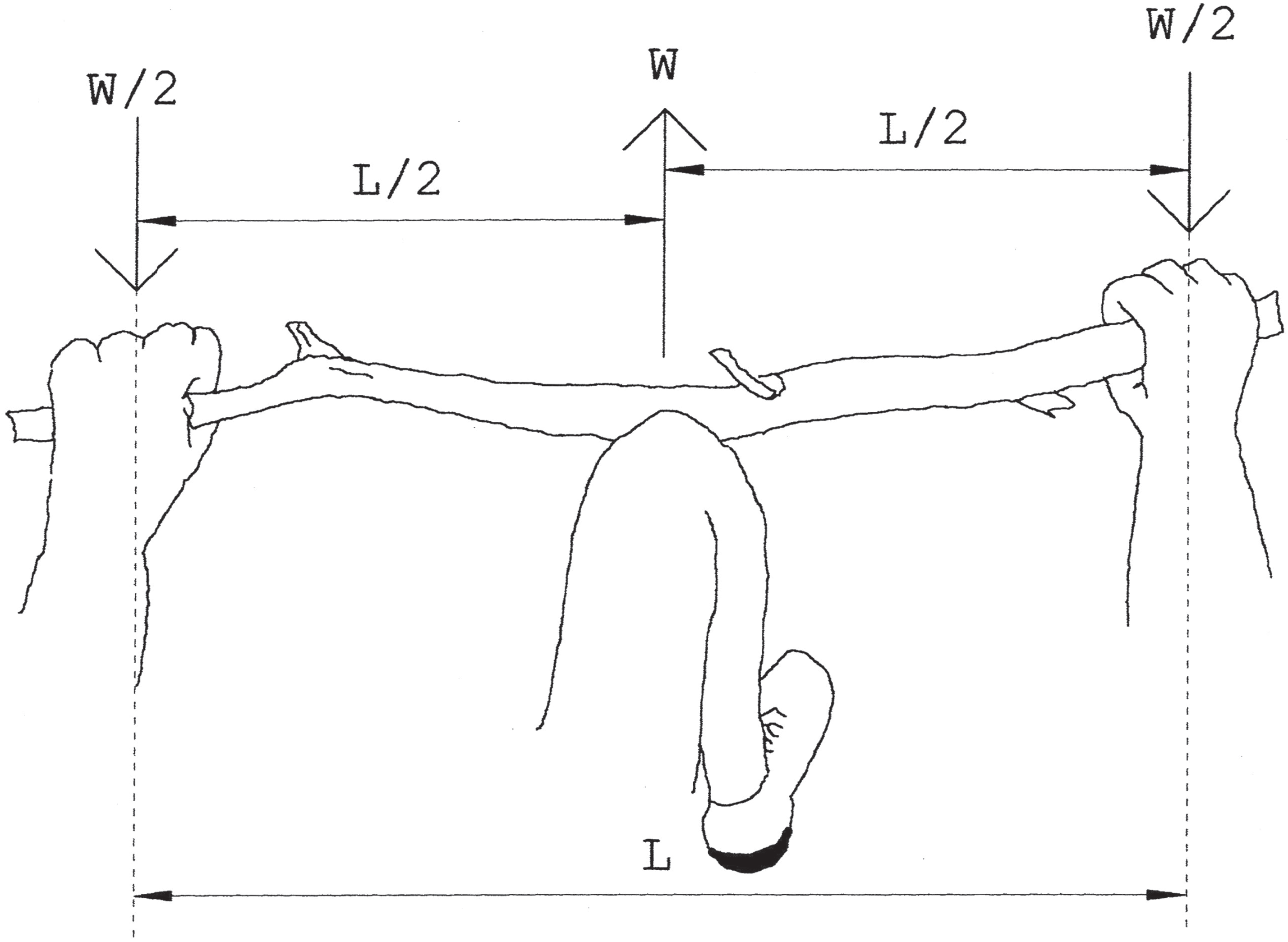
For an example which will be closer to our area of interest consider the bending or breaking of a branch, as in Fig. 3-4. The branch is held with our hands as far apart as possible and is pulled against our knee, which is placed midway between our hands. The outward push by our knee and the inward pull by our hands are examples of forces which are parallel to each other but act in opposite directions. Such a combination of forces is said to comprise a *couple* and these are vital elements in understanding the internal workings of beams.

If L is the distance between our hands and W the force against the branch by our knee, the pull by each hand must be W/2. The moment M of the force seeking to bend or break the branch will be W/2 acting through its lever arm L/2, the distance between our hand and our knee.

So we have: $M = W/2 \bullet L/2 = WL/4$

Fig. 3-4

Breaking A Branch



The centrally-loaded beam in Fig. 3–5 is analogous to the bent branch in Fig. 3–4. As before, the combination of force and lever arm attempting to bend the beam is either reaction (W/2) multiplied by L/2, its distance from the load. The maximum bending moment is:

$$M = WL/4 = \frac{2000\ \text{lb} \times 180\ \text{in}}{4} = 90{,}000\ \text{in-lb}$$

In beams which are supported at both ends, concentrating the load at the midpoint produces the greatest possible bending moment. In considering a large movable load one must assume that it will, on some occasion, be placed at the beam midpoint and design accordingly.

In order to explain single-load bending moment calculations it has been necessary, in this and the succeeding diagram (Fig. 3–6), to neglect the weight of the beam. In practice, this may be permissible with wood but not with concrete beams. Also, where beams are not fixed to their supports, L is longer than the clear span separation shown in these diagrams. The following rule applies to bending moment calculations for beams with single loads:

"The bending moment at any point along a beam carrying a single concentrated load can be determined by multiplying the reaction at a support by the distance between the chosen point and the support".

In Fig. 3-6, a beam across a 200" span carries a single 2000 lb weight 60" from its left support. We begin by calculating the reactions at the supports:

$$R_1 = \frac{2000\ \text{lb} \times 140\ \text{in}}{200\ \text{in}} = 1400\ \text{lb} \qquad R_2 = 600\ \text{lb}$$

The bending moment at point D, where the weight is concentrated is:

$$M = R_1\ (1400\ \text{lb}) \times 60\ \text{in} = 84{,}000\ \text{in-lb}$$

$$\text{Or } M = R_2\ (600\ \text{lb}) \times 140\ \text{in} = 84{,}000\ \text{in-lb}$$

If the weight were moved to the mid-point of the beam the maximum bending moment would increase to 100,000 in-lb.

Fig. 3-5

Bending Moment: Central Concentrated Load

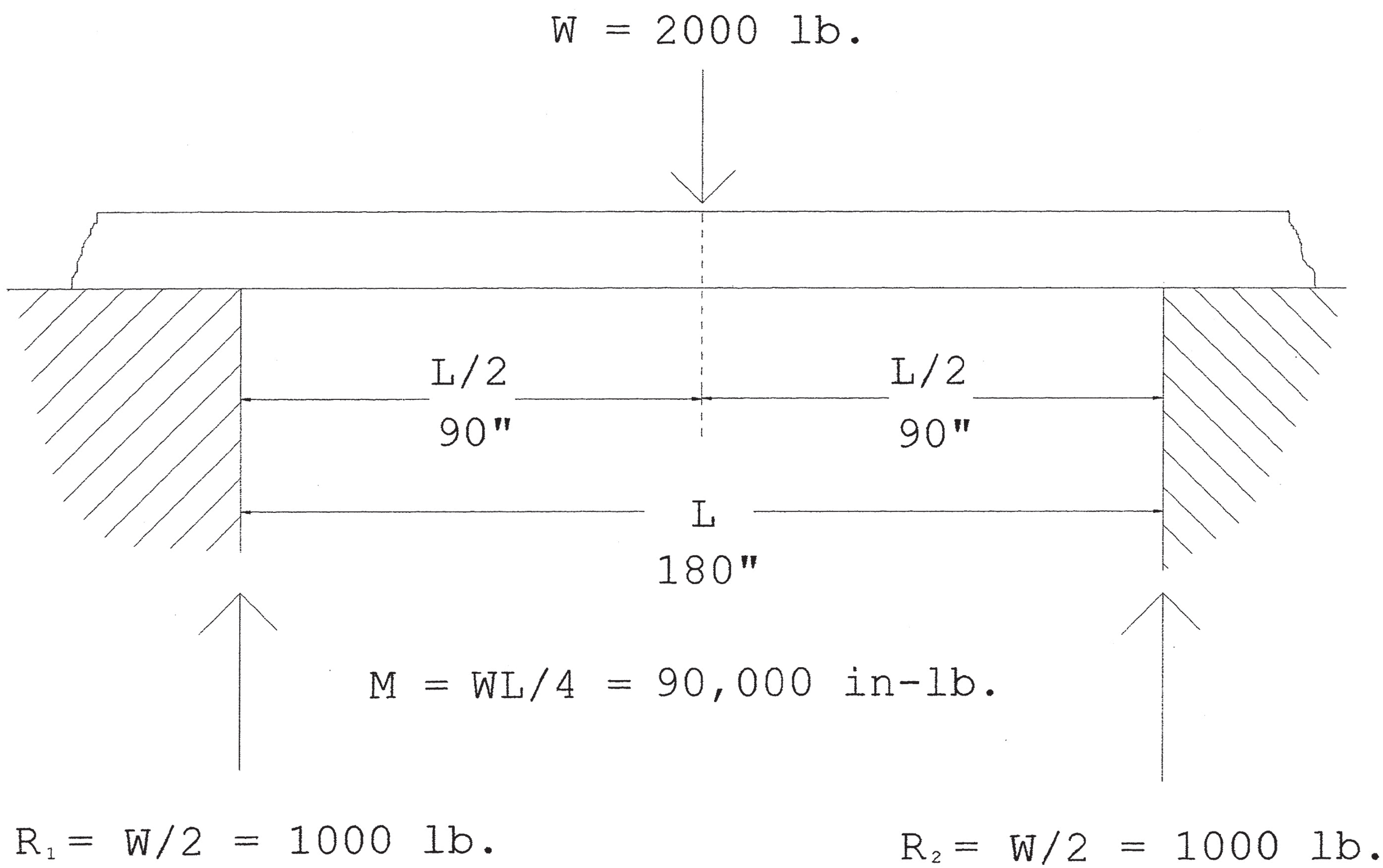
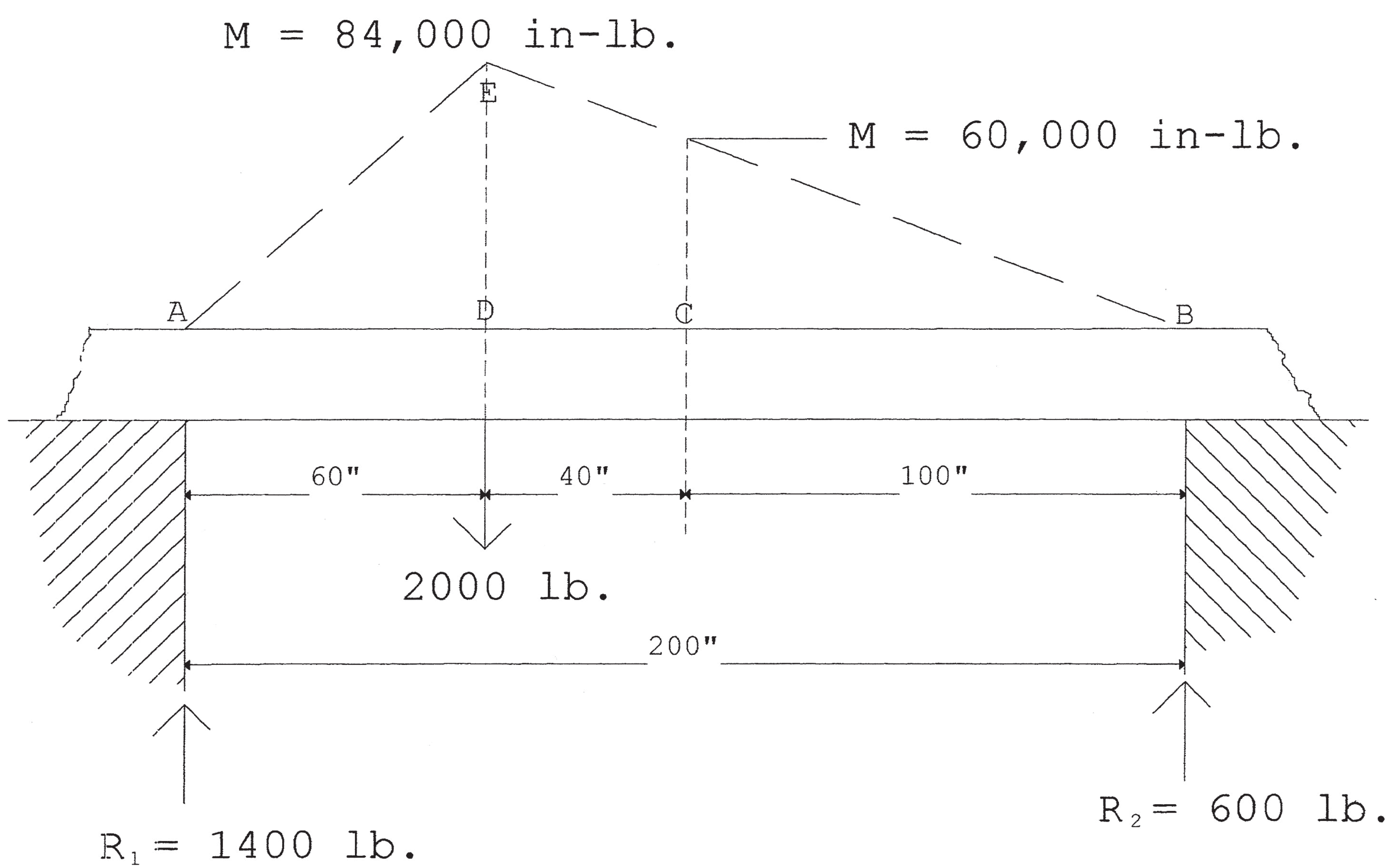


Fig. 3-6

Bending Moment: Single Concentrated Load



Suppose we were not convinced that the maximum bending moment occurred at point D and we wished to determine its value at the midpoint of the beam, point C. Here we have:

$$M = R_2\ (600\ \text{lb}) \times 100\ \text{in} = 60{,}000\ \text{in-lb}$$

The dashed lines AE and BE show how the bending moment varies over the length of the beam. There is, however, one important limitation to these single-load calculations. Bending moments between points A and D can only be calculated from the reaction R_1, and those between B and D from reaction R_2, as was done above. Calculations from either support must end at the weight (point D), for reasons which will be explained in the following section.

The object of the preceding exercises, with single loads and weightless beams, was to prepare the reader for the practical descriptions to follow. These involve bending moment calculations for any point along beams carrying multiple weights. We begin by stating the rule governing these calculations, the first part of which will be familiar:

"The bending moment at any point along a beam carrying one or more concentrated loads can be found by multiplying <u>either reaction</u> by its distance to the point where the bending moment is to be found, but if any weights lie between the selected reaction and that point, a deduction must be made equal to the sum of the weights multiplied by their respective distances from the point at which the bending moment is to be found."

The above statement seems complicated, but its application is fairly simple, as the following example will show. We can illustrate its use by calculating the bending moment at the midpoint (point C) of the Fig. 3–6 beam from the left-side reaction R_1. The diagram shows that R_1 presses upward with a force of 1400 lb and acts upon point C through its lever arm AC. However, lying upon this lever arm we have a 2000 lb weight which presses in the opposite (downward) direction to R_1 and acts upon point C through its lever arm DC. The bending moment at point C will be the difference between the two opposing effects.

$$M = R_1 \bullet AC - 2000\ \text{lb} \bullet DC$$

$$= 1400\ \text{lb} \times 100\ \text{in} - 2000\ \text{lb} \times 40\ \text{in} = 60{,}000\ \text{in-lb}$$

We are now able to calculate the maximum bending moment of uniformly loaded members, which will probably make up the vast majority of the design pieces within any small building project. Possible examples include elevated slabs carrying randomly-placed loads, a beam supporting such a slab, or a combination of members supporting the roof of an earth-sheltered house. In Fig. 3–7 the uniform load is represented by concrete blocks evenly distributed over the entire span L of a beam. In reality, such a load would be recalculated to include the weight of the beam. According to our previous calculations, the maximum bending moment will be at B, the beam midpoint. Calculating from the left-side support, the reaction R_1 which has a magnitude of W/2, acts upon point B through its lever arm AB which is equal to L/2 in length. Hence the upward action of R_1 on point B will be:

W/2 • L/2 = WL/4 as we have previously found

However, the load section which lies between points A and B and has its centre of gravity at its midpoint C presses downward upon the lever arm AB with a weight of W/2, and acts upon point B through its lever arm CB, which is equal to L/4. The maximum bending moment (at point B) of the uniformly loaded beam will be:

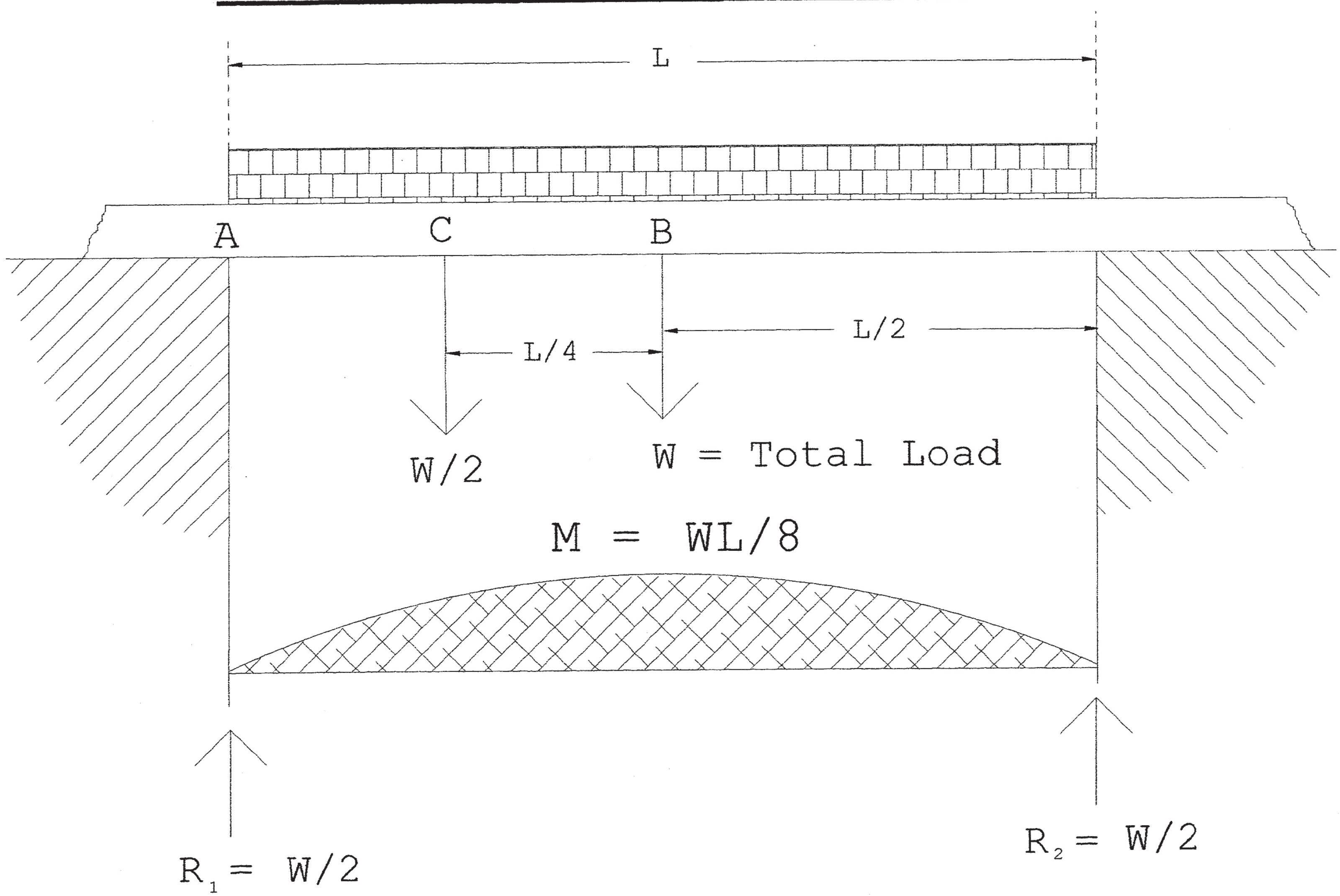
M = WL/4 - W/2 • L/4 = WL/4 - WL/8 = WL/8

Along beams carrying a uniformly distributed load, the bending moment variation follows a curve instead of the straight line seen in Fig. 3-6. This curvature does not affect our calculation of the maximum bending moment at the beam centre.

Although modifications of the M = WL/8 bending moment formula will meet most small project design requirements, some exceptions can be expected. In Winterholm, half of the upper level double-garage floor is an elevated slab, which is also the ceiling of a storage room/pantry. The slab and its rebar were sized according to bending moments which might result from the entry of a fully loaded truck, with an assumed load of 2000 lb upon each of its rear wheels.

Fig. 3-7

Bending Moment: Evenly Distributed Load



Although we are now able to calculate the bending moment at any point along a beam, if we are dealing with multiple or irregularly-spaced loads, as in Fig. 3-2, the location of the point of maximum bending moment along the beam must be determined. This could be done by trial-and-error, making bending moment calculations at different locations until the largest value is found. A far better approach is based upon the relationship between bending moment and vertical shear in beams. At the end of the following section a beam carrying multiple loads is analyzed by a combination of bending moment and shear calculations.

Shear

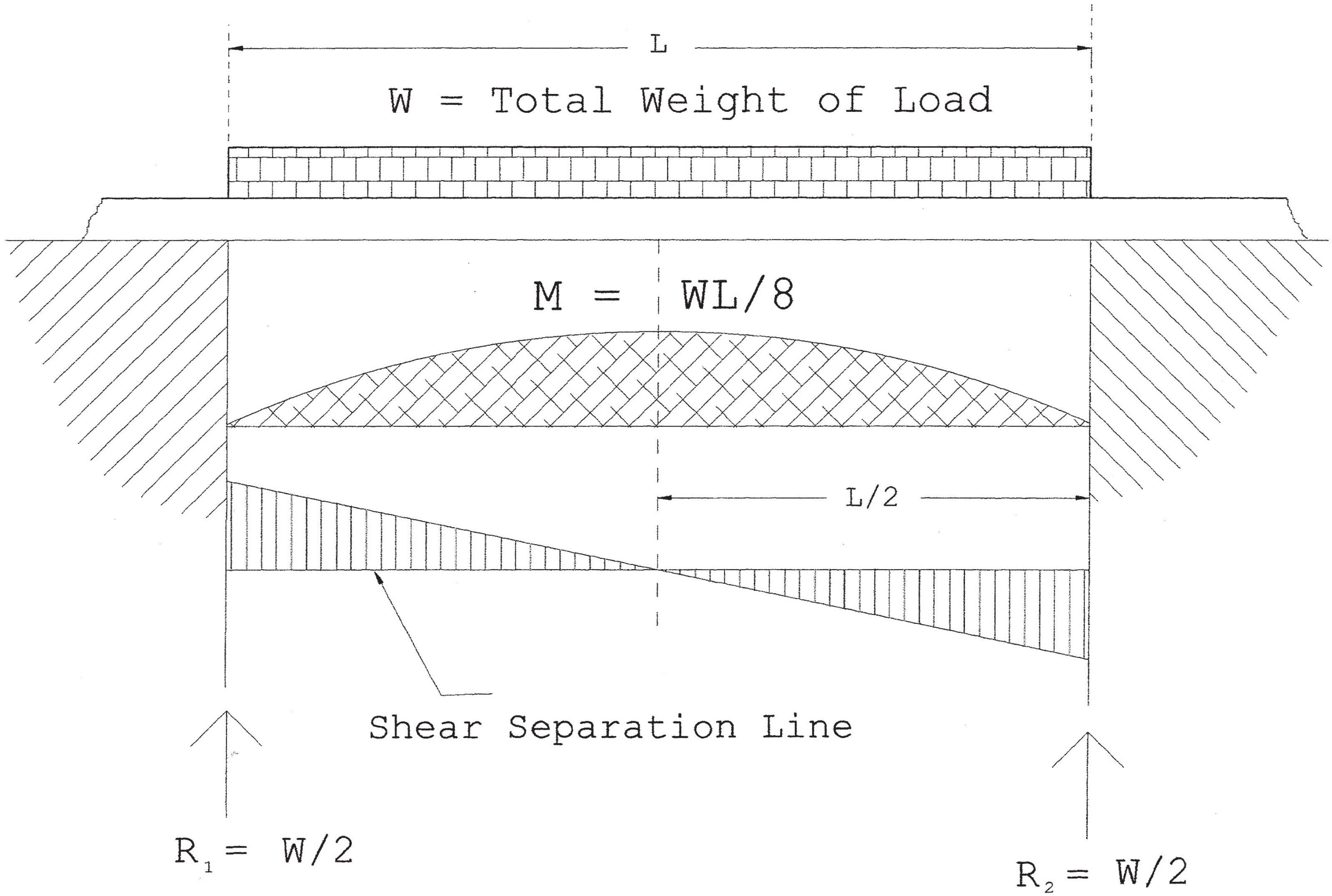
Two types of shear are developed within loaded beams. One can imagine a beam composed of thin horizontal layers. If such a beam was bent downward there would be a tendency for these layers to slip relative to each other, which would constitute horizontal shear. If, on the other hand, we imagine a beam composed of blocks with vertical joints, the load would tend to slide the blocks downward relative to the beam supports. This is vertical shear, and it is this type of shear which primarily concerns us. The amount of shearing stress depends upon the size and distribution of the load, and is independent of the beam length.

At any point along a beam, the vertical shear will be the difference between the sizes of the upward and downward forces at that point. Vertical shear will reach a maximum value at the beam supports. In unequally-loaded beams, the maximum vertical shear will be equal to the size of the largest reaction, and the beam must be designed to accommodate this stress. This section deals only with the variations in vertical shear along beams. The physical effect of shear on beams and slabs will be discussed in Chapter 5.

Diagrams showing the variations in vertical shear along beams can be constructed as follows: a horizontal *shear separation line* is drawn along the length of the beam. Shear stresses relating to loads supported by the left side reaction are placed above this line, and those relating to loads supported by the right side reaction are shown as negative quantities below the separation line. As an

Fig. 3-8

Bending Moment & Shear: Evenly Distributed Load



example, let us redraw Fig. 3–7 and show the distribution of both vertical shear and bending moment. Fig. 3-8 shows, as expected, the shear reaching a maximum at both supports. Minimum shear occurs at the beam midpoint, a distribution opposite to that of the bending moment. This illustrates an important rule:

"The bending moment will reach a maximum value wherever the vertical shear passes through zero i.e., crosses the shear separation line."

The vertical shear at any point along a beam will be the supported weight remaining between that point and the intersection point along the shear separation line. In practice, calculations begin at one of the supports. Here shear is at a maximum, equal to the total reaction. However, as we move along the beam, this maximum shear is decreased by whatever weight lies between our chosen point and the support. The evenly distributed load in Fig. 3–8 produced a linear decrease in shear away from the supports.

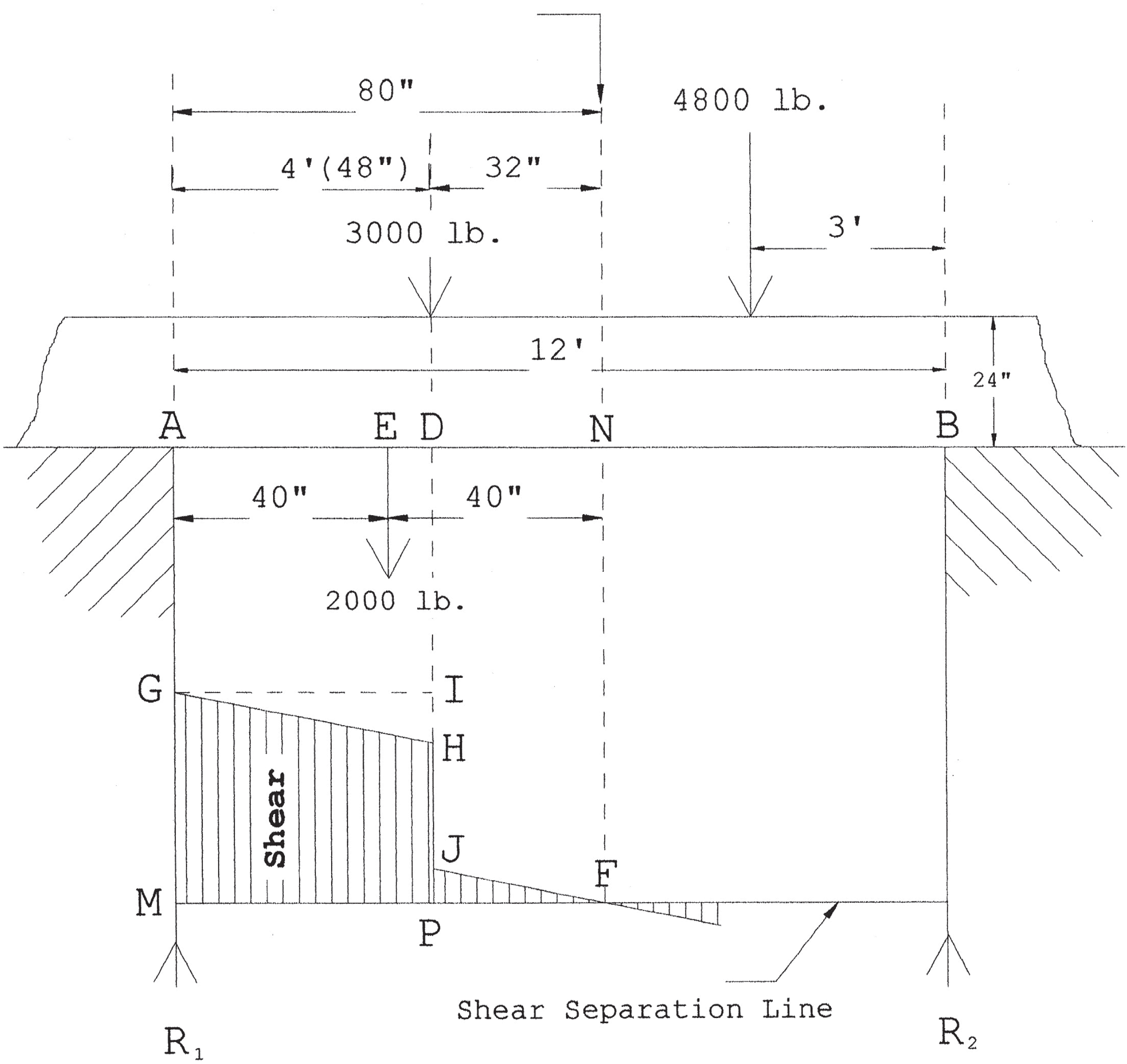
In Fig. 3–9, a reinforced concrete beam 12″ wide and 24″ deep lies across a 12′ span and carries two concentrated loads: 3000 lb at 4′ from the left support, and 4800 lb at 3′ from the right support. In order to design the reinforcement for this beam we must know the maximum vertical shear in the concrete and the maximum bending moment.

The first part is relatively straightforward. We know that shear is greatest at the supports, so the problem is simply to determine which reaction is largest and its value will be equal to the maximum shear within the beam. In Fig. 3–9, the concentrated loads are partitioned between the two reactions according to the rules given in the Reactions section of this chapter. The weight of the concrete beam is divided equally between the reactions. The density of reinforced "normal" concrete is assumed to be 150 lb/ft^3. The unsupported section of the beam contains 24 ft^3 of concrete so it weighs 150 x 24 = 3600 lb. The maximum vertical shear is 6400 lb, the size of the R_2 reaction. The sections of rebar which are cast lengthwise into beams to provide tensile strength to the concrete do not contribute to the beam's resistance to shear stresses. Additional *web* reinforcement, described in Chapter 5, is used for this purpose.

Fig. 3-9

Maximum Bending Moment: Multiple Loads

Maximum Bending Moment (Pt. N) = 224,000 in-lb.



1800 lb.	Weight of Beam	1800 lb.
2000 lb.	3000 lb. Load	1000 lb.
1200 lb.	4800 lb. Load	3600 lb.
5000 lb. = R_1	**Reactions**	R_2 = 6400 lb.

The more difficult determination of the maximum bending moment is made in two stages. First, the point along the beam where the bending moment is greatest must be located, and then its value at that point must be calculated. As previously stated, the point of maximum bending moment will be found by locating the point of zero vertical shear.

Although we can work from either reaction, it is more convenient to begin at the left support and work with positive values. Point M is chosen arbitrarily as the position of the shear separation line upon which the vertical shear distribution will be plotted. From point M a length, MG, is scaled off vertically, representing the shear at the reaction R_1, which is 5000 lb. As we move along the beam and away from the support, the shear will be decreased by whatever weight lies between the point chosen and the support. The initial decrease in shear is caused by the weight of the concrete beam. The beam weighs 3600 lb and is 12′ (144″) long, so every inch of beam weighs 3600 ÷ 144 = 25 lb. For every inch we move along the beam, away from point A, the shear will decrease by 25 lb. Upon reaching point D, which is 48″ from point A, the shear will have decreased by 48 x 25 = 1200 lb, which is represented by the distance HI. The straight-line decrease in shear as shown by the line GH is characteristic of evenly distributed loads and is also evident in Fig. 3-8. At point D we find an isolated 3000 lb load which is represented by the distance HJ. The shear remaining at point D is 5000 – (1200 + 3000) = 800 lb represented by JP. Beyond point D, the weight supported by reaction R_1 continues to decrease by 25 lb/in because of the concrete beam. The distance to the zero-shear crossover point DN, is 800 ÷ 25 = 32″. The point of zero shear and maximum bending moment is N, 80″ from the support at A. One could complete the shear diagram by adding negative weights beyond point F. A sum of -6400 lb (R_2) at point B would confirm that the calculations were correct.

To calculate the bending moment at point N we begin with the reaction R_1 which presses upward with a force of 5000 lb and acts upon point N through its 80″ lever arm AN.

We have: 5000 lb x 80 in = 400,000 in-lb

However, lying upon this lever arm are two loads, both pressing downward, in opposition to R_1. The first of these